

# Progression In

# **Written Methods**

# Of Calculation At St Bede's

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#### PROGRESSION IN CALCULATION

#### From Foundation Stage to Year 6

The aim for written calculations is different from the aim for mental calculations. With mental work, the aim is to teach children a repertoire of strategies from which to select. With written calculations the ultimate aim is proficiency in a compact method for each operation.

#### Mental calculation

Strategies for mental calculation are introduced from Y1 or Y2 to Y3 and developed further in Y4, Y5 and Y6. All children, apart from those with significant special educational needs, should be introduced to the full range of mental calculation strategies when they have the necessary pre-requisite skills. Children with significant special needs should learn a narrow range of strategies which are generally applicable.

#### Written calculations

Building on the mental strategies they have used so that they can understand the process, children need first to be taught to record their methods in an expanded form. When ready, they are taught how to refine the recording to make it more compact.

The methods and layouts to be taught for each operation are detailed in this document.

#### Challenges to teachers

- Ensuring that recall skills are established first so children can concentrate on a written method without reverting to first principles
- Making sure that, once written methods are introduced, children continue to look out for and recognise the special cases that can be done mentally;
- Catering for children who progress at different rates; some may grasp a compact method of calculation while others may never do so without considerable help;
- Catering for children who can carry out some standard methods successfully, e.g. for +, but not -;
- Recognising that children tend to forget a standard method if they have no understanding of what they
  are doing

Often the compactness of a vertical method conceals how mathematical principles are applied, e.g. children may use place value when working mentally, but be confused in written work because they do not understand how place value relates to 'carrying'. There can be long-lasting problems for those taught compact, vertical methods before they understand what they are doing.

Simply correcting children's errors may help in the short-term, but not permanently. They need to understand why a particular method works rather than simple following a set of rules. They can then fall back to a simpler method if uncertain or to check their answer.

**NOTE:** In the following guidance, suggestions are given as to when written methods and particular layouts should be introduced. However, the most importance thing to consider rather than children's age, is whether they have the necessary pre-requisite skills.

Hundreds tens and units are now known as Hundreds tens and ones

### Progression towards a written method for Addition

Before the introduction of formal written methods for addition, children should be able to:

- 1. recall all addition pairs to 9 + 9 and complements in 10;
- 2. add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- 3. add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- 4. partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

#### **EARLY STAGES (EYFS)**

Children will engage in a wide variety of songs and rhymes, games and activities.

They will begin to relate addition to combining two groups of objects, first by counting all and then by counting on from the largest number.

They will find one more than a given number.

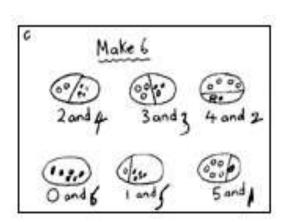
In practical activities and through discussion they will begin to use the vocabulary involved in addition.





'You have five apples and I have three apples. How many apples are there altogether?'

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



#### Statutory requirements

Pupils should be taught to:

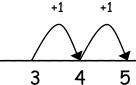
- read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- represent and use number bonds within 20
- add one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as  $7 = \square + 5$ .

Children use number tracks, numberlines and practical resources, to support calculation.



They count on from the largest number.

3 + 2 = 5



2 0 1

6 7 8 9

#### **Statutory requirements**

Pupils should be taught to:

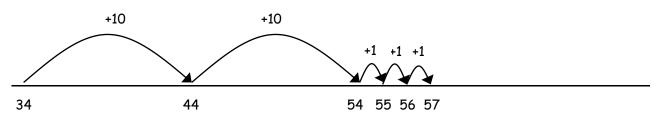
add numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

First counting on in tens and ones.

$$34 + 23 = 57$$



Children then progress to more efficient methods by adding units in one jump and then tens in one jump.

When adding two digit numbers the children can use partitioning which can help with mental calculations too.

$$26 + 43 =$$
 $20 + 40 = 60$ 
 $6 + 3 = 9$ 
 $60 + 9 = 69$ 

In Y2 the children are also introduced to the column addition method, which will also support their understanding of place value.

26

<u>43</u> +

69

#### **Statutory requirements**

Pupils should be taught to:

add numbers with up to three digits, using formal written methods of columnar addition

Build up starting with 2 digits + 2 digits

#### Introducing the compact layout

- no carrying (demonstration stage only), e.g. 54 + 35, 326 + 271
- carrying from ones to tens, e.g. 47 + 26, 368 + 423
- carrying from tens to hundreds, e.g. 368 + 481
- carrying from ones to tens and tens to hundreds, e.g. 47 + 76, 368 + 478
- a mixture of 'carries'

E.g. demonstrate the method children currently use alongside the new method. Children should then practise the new method. (3 6 8 + 4 7 8)

When the compact layout is introduced, the language of place value should continue to be used but when children are confident, they will use 'digit-speak', e.g. for the addition of two three-digit numbers above, they are likely to say:

- 8 add 8 is sixteen; 6 in the answer and carry 1
- 6 add 7 is 13 plus the carry 1 is 14; 4 in the answer and carry 1
- 3 add 4 is 7 plus the carry 1 is 8
- Answer is 846

This is a form of shorthand that speeds up the process of addition. If children consistently carry out a range of calculations correctly, it is likely that they understand the process. However, teachers should occasionally check their understanding by asking children to explain what exactly they mean at each stage of the calculation, e.g. what does that 'carry 1' really mean?

#### Statutory requirements

Pupils should be taught to:

 add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate

See the column method shown for Y3

Children in Y4 will meet some fairly simple additions of £.p. some children may be able to use the compact column method, but those who have not learnt how to use this method should change pounds to pence and add using a compact method.

Ensure that children are aware that the decimal points should line up especially when using mixed amounts. Eg £3.59 + 78p =

3.59 0.78 +

4.37

#### YEAR 5

#### Statutory requirements

Pupils should be taught to:

 add whole numbers with more than 4 digits, including using formal written methods (columnar addition)

Continue column addition method taught in earlier year groups.

These should be extended through addition of money and measures.

#### YEAR 6

There are no formal statutory requirements for addition in Y6. They are expected to continue with the formal written methods and apply this to problem solving.

## Progression towards a written method for **Subtraction**

#### **EARLY STAGES (EYFS)**

Children will engage in a variety of counting songs and rhymes and practical activities.

In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.

They will begin to relate subtraction to 'taking away' using objects to count 'how many are left' after some have been taken away.

6 - 2 = 4



Take two apples away. How many are left?'

Children will begin to count back from a given number.

#### YEAR 1

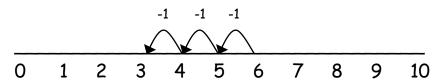
#### **Statutory requirements**

Pupils should be taught to:

- read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs
- subtract one-digit and two-digit numbers to 20, including zero

The numberline should be used as the children will be using this for addition too. Numberlines can be used as scaffolds but **children should also create own numberlines.** 

$$6 - 3 = 3$$



The numberline should also be used to show that 6 - 3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart. Children count backwards.

#### **Statutory requirements**

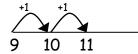
Pupils should be taught to:

- subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers

Children can continue to use a number line counting back and counting on when finding small differences.

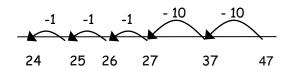
In Y2 Children should be taught the counting on method, using a number line; this will establish the relationship between the operations of addition and subtraction.

11 - 9 = 2



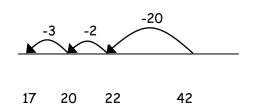
First, counting back in ones then progressing to tens and ones and then beyond.

47 - 23 = 24



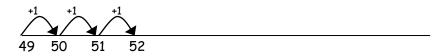
Bridging through ten can help children become more efficient.

42 - 25 = 17



Counting on to find a small difference

52 - 49 = 3



By Spring, introduce column method and use equipment (YPO and place value counters). Children should be taught to use the column subtraction method without 'carrying'. This supports place value and prepares for formal written methods with larger numbers. **More able childrenn can exchange** 

#### YEAR 3

#### **Statutory requirements**

Pupils should be taught to:

subtract numbers with up to three digits, using formal written methods of columnar subtraction

Children should be confident in carrying out subtraction by counting back or on, using a numberline, before being introduced to the formal method of decomposition. Some children find decomposition difficult. **Use** efficient methods not expanded method

#### **Decomposition**

Before the introduction of the formal written method for subtraction (decomposition), children should be able to:

- 1. recall all subtraction facts to 20;
- 2. subtract multiples of 10 (such as 160 70) using the related subtraction fact, 16 7, and their knowledge of place value;
- 3. partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Teach the decomposition method in this order:-

- 1. TU TU, then HTU TU and HTU HTU, exchange from tens to units, e.g. 71 46, 173 38, 774 248
- 2. HTU HTU, exchange from hundreds to tens, e.g. 553 272
- 3. HTU HTU, exchange from tens to units and from hundreds to tens, e.g. 635 278

$$71 - 46 =$$

This should be formally recorded.

Y3 onwards decomposition and build up to then apply to word problems

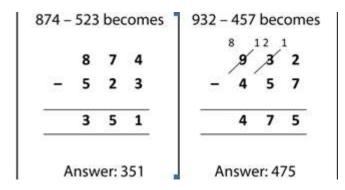
#### Statutory requirements

Pupils should be taught to:

 subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate

Countinue, or begin, with the formal written method of decomposition as set out in Y3. Extending this to

- Subtract numbers with up to four digits, including numbers with different numbers of digits
- Subtraction with numbers involving zeros e.g 5001 2345
- Subtract decimals, money and measures



#### YEAR 5

#### Statutory requirements

Pupils should be taught to:

 add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

Follow column subtraction (decomposition) method taught in earlier year groups.

#### YEAR 6

There are no formal statutory requirements for subtraction in Y6. They are expected to continue with the formal written methods and apply this to problem solving.

**NB** Don't always use a formal sum if it is more sensible to work it out mentally or using a numberline!! eg 2008-698

## Progression towards a written method for Multiplication

#### **EARLY STAGES (EYFS)**

Children will engage in a wide variety of songs and rhymes, games and activities.

In practical activities and through discussion they will begin to solve problems involving doubling. Lots of concrete and pictorial





'Three apples for you and three apples for me. How many apples altogether?'

#### YEAR 1

#### Statutory requirements

Pupils should be taught to:

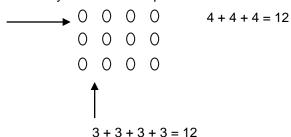
- solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Pictorial recording as children develop understanding of repeated addition, e.g. counting in sets of two, e.g. sets of two pence coins, five, e.g. tally bundles, tens, e.g. sets of 10 pennies. Children will use a number line or repeated addition. Only some children will use a number line.



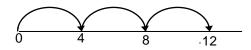




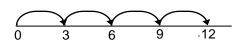
• Use of arrays to illustrate repeated addition.



Record repeated addition on a number line
 e.g. for the array above,



4 + 4 + 4 = 12 leading to four, three times or  $4 \times 3$ 



3 + 3 + 3 + 3 = 12 leading to three, four times or  $3 \times 4$ 

Using such models will help develop children's understanding of the commutativity of multiplication, e.g.  $6 \times 2 = 2 \times 6$ 

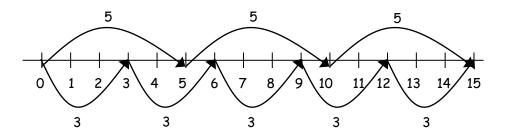
#### **Statutory requirements**

Pupils should be taught to:

- recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (x) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative)

Children should continue with the numberline, to show repeated addition, and commutative. Take out the numberline keep then Just use arrays and partitioning?

Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.



Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

#### Year 3 to Year 6

Before the introduction of formal written methods for multiplication, children should be able to:

- recall multiplication facts for the tables used;
- partition numbers into multiples of one hundred, ten and one;
- work out products such as 70 x 5, 70 x 50, 700 x 5 or 700 x 50 using the related fact 7 x 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7,
   and their knowledge of place value;
- add combinations of whole numbers using the column method.

#### Statutory requirements

Pupils should be taught to:

- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they
  know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal
  written methods

Year 3 use the grid method with tables children are confident with

<u>Grid method</u> Use grid in first term but do short multiplication method by Summer. Only use the tables that Y3 work on in these calculations

TU x U

(Short multiplication - multiplication by a single digit)

23 x 8

Children will approximate first 23 x 8 is approximately 25 x 8 = 200

#### **Short multiplication**

• TU x U, e.g. 23 x 8 Demonstrate alongside the grid layout. Children use short multiplication.

#### **Statutory requirements**

Pupils should be taught to:

multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Children should progress to using the formal written layout for multiplication problems.

24 × 6 becomes

2 4 × 6 1 4 4

Answer: 144

 $342 \times 7$  becomes

\* 7 2 3 9 4

Answer: 2394

This can also be extended to include money £.p x U, e.g. £3. 64 x 8

**Note:** Children who are continuing to use the grid layout should, when working with money or measures, convert to the smaller measure e.g. convert £.p to pounds, and then back to the original units if required.

#### Statutory requirements

Pupils should be taught to:

- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply whole numbers and those involving decimals by 10, 100 and 1000

#### **Short Multiplication**

 $24 \times 6$  becomes

Answer: 144

 $342 \times 7$  becomes

Answer: 2394

2741 × 6 becomes

Answer: 16 446

#### Long multiplication-

<u>Long Multiplication</u> use efficient method not grid (unless child really can't do)

NB We multiply the units first and we carry the number on the doorstep.

24 × 16 becomes

Answer: 384

124 × 26 becomes

Answer: 3224

#### **Statutory requirements**

Pupils should be taught to:

 multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Children should apply the formal written methods taught in earlier year groups to more complex numbers, including decimals and measures.

NB Make sure children know multiplication can always be done in any order!

## Progression towards a written method for Division

#### **EARLY STAGES (EYFS)**

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving halving and sharing.

Share the apples between two people.





'Half of the apples for you and half of the apples for me.'

#### YEAR 1

#### **Statutory requirements**

Pupils should be taught to:

- solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Use **sharing** to answer division questions such as: Share this bag of 12 counters into 3 pots.



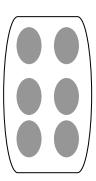


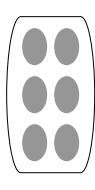


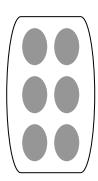
• Experience division as grouping, such as:

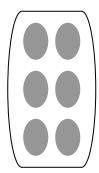
24 eggs are packed in boxes of 6. How many boxes are needed?

Take 6 eggs and pack the first box. Continue until there are no eggs left. Then count how many boxes have been used, recording pictorially.









#### **Statutory requirements**

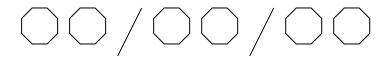
Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for division and write them using the division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

In Y2 we teach and model division but children only use concrete or pictorial?

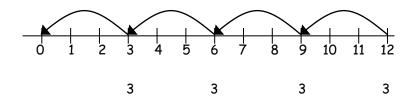
Introduce the number sentence and allow children to use sharing or grouping pictorial representations to solve.

There are 6 sweets, how many people can have 2 sweets each?  $6 \div 2 = 3$ 



Repeated subtraction using a number line or bead bar

$$12 \div 3 = 4$$



Using symbols to stand for unknown numbers to complete equations using inverse operations

Experience divisions that give rise to **remainders**, such as, in a sharing context:

Three friends share 17 marbles equally. How many marbles does each friend get? How many marbles

are left over?









19

 $17 \div 3 =$ 

They get 5 marbles each and there are 2 left over.

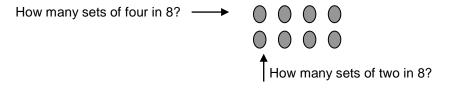
..and in the context of grouping:

Grandma is sending Christmas cards. She needs 2 stamps to stick on each envelope. She has 11 stamps, how many cards can she send?



She can send 5 cards and has 1 stamp left over.

Relate division to multiplication using arrays.



#### Year 3 to Year 6

To progress towards short division, children need to be able to:

- understand and use the vocabulary of division;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts for the tables used;
- recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally, e.g. find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

#### **Statutory requirements**

Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for division using mental and progressing to formal written methods

#### **Expanded layout for short division**

Model this to support understanding the place value within the compact method but children are not expected to record this method. Record in compact form ( short division)

There is No formal requirement for Y3/Y4 to do division, but in St Bede's we will start short division in Y3 with no carrying or remainders (High ability can do remainders)

#### Stage 1

TU ÷ U, no remainder and no carrying, e.g. 69 ÷ 3

$$69 \div 3 = 23$$

#### Stage 2

TU ÷ U, remainder but no carrying, e.g. 67 ÷ 3

$$67 \div 3 = 22 R 1$$

$$92 \div 4 = 23$$

#### Stage 3 (compact division)

TU ÷ U, no remainder and no carrying, e.g. 93 ÷ 3

30 threes in 90 and 1 three in 3. The answer is 31.

#### **Statutory requirements**

There are no formal statutory requirements linked to division in Year 4.

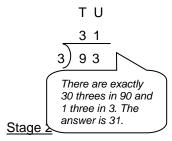
Children should practise and consolidate the division work in Year 3. Extended by introducing the compact short division method with remainders. Aim for 3 digit and remainders/carrying by end of Y4

#### Compact layout for short division

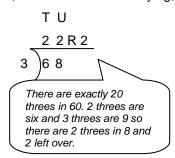
Initially, children will continue to use correct place value vocabulary, but as they become more confident, they can be introduced to 'digit-speak'. Dialogue with place value vocabulary is illustrated for Stages 1-4 and 'digit-speak' shown for Stage 4.

#### Stage 1

TU ÷ U, no remainder and no carrying, e.g. 93 ÷ 3

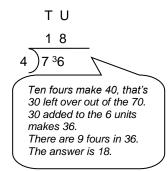


TU ÷ U, remainder but no carrying, e.g. 68 ÷ 3



#### Stage 3

TU  $\div$  U, carrying from T to U but no remainder, e.g. 76  $\div$  4. When dealing with carrying figures, relate to knowledge of place value.



#### Stage 4

 $TU \div U$  with carrying and remainder, e.g.  $96 \div 7$ 

Ten sevens make 70, that's 20 left over out of the 90. 20 added to the 6 units makes 26.

Three sevens are 21, that's 5 left over out of the 26. The answer is 13 R 5

'Digit-speak' version
How many sevens in 9?
Once 7 is 7, so that's 1 and 2 (tens) left over.
How many sevens in 26?
Three sevens are 21, so that's 3 and 5 left over.
The answer is 13 R 5.

#### **Statutory requirements**

Pupils should be taught to:

 divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Continue to develop the short division method as follows:-

Extending the compact layout for short division

Order of difficulty of calculations:

- 1. No remainder, no carrying, e.g. 844 ÷ 4
- 2. Remainder, no carrying, e.g. 486 ÷ 4
- 3. No remainder, carrying from T to U, e.g. 860 ÷ 4
- 4. No remainder, carrying from H to T, e.g. 928 ÷ 4
- 5. No remainder, carrying from H to T and T to U, e.g. 984 ÷ 4
- 6. Remainder and carrying, e.g. 743 ÷ 4
- 7. Examples where consideration needs to be given to the placing of the quotient,

$$\frac{0\ 9\ 6}{13^{38}}$$
 r 3

Initially pupils should cross out the hundreds digit and carry it over to the tens as well as placing zero in the quotient.

8. Examples where there are zeros in the quotient, e.g.  $818 \div 4$ ,  $5609 \div 8$ 

$$\frac{2\ 0\ 4}{18}$$
 r 2

Emphasise zero as place holder.

9. Express remainders as fractions, e.g. 387 ÷ 4

$$\frac{0 \ 9 \ 6}{4 \ ) \ 3^{3}8^{2}7}^{34}$$

- 10. Decimals
- 11. Measures
- 12. Divide amounts of money,
  - e.g. Grandma emptied her money box. There was £12.46. She shared it equally between her five grandchildren. How much did they each get and how much was left over?

$$\frac{0\ 2\cdot 4\ 9}{5) \cancel{x}^{12} \cdot {}^{24} \cdot {}^{46}}$$
 r 1

Each child got £2.49 and there was 1p left over.

13. Varied contexts where the remainder is expressed as a whole number, or a fraction, or a decimal or where the quotient needs to be rounded up (e.g. 72 children are going camping. Each tent holds 5 children. How many tents do they need?), or down (The farmer has collected 91 eggs. Egg boxes each hold 6 eggs. How many boxes can he fill?).

YEAR 6 Children should use the short division method to work out long division (in preparation for Carmel). Chunking method can also be used

#### **Statutory requirements**

Pupils should be taught to:

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

#### **Short Division**

98 ÷ 7 becomes

1 4 7 9 8

Answer: 14

432 ÷ 5 becomes

8 6 r 2 5 4 3 2

Answer: 86 remainder 2

496 ÷ 11 becomes

4 5 r1 1 1 4 9 6

Answer:  $45\frac{1}{11}$ 

#### **Long Division**

432 ÷ 15 becomes

2 8 r 12
1 5 4 3 2
3 0 0
1 3 2
1 2 0
1 2

Answer: 28 remainder 12

432 ÷ 15 becomes

Answer:  $28\frac{4}{5}$ 

432 ÷ 15 becomes

Answer: 28.8